SR2004: DESIGN YOUR OWN UNDULATOR

Allan Baldwin

Diamond Light Source

Rutherford Appleton Laboratory

TUTORIAL OUTLINE

THE TUTORIAL HAS THE FOLLOWING STRUCTURE:

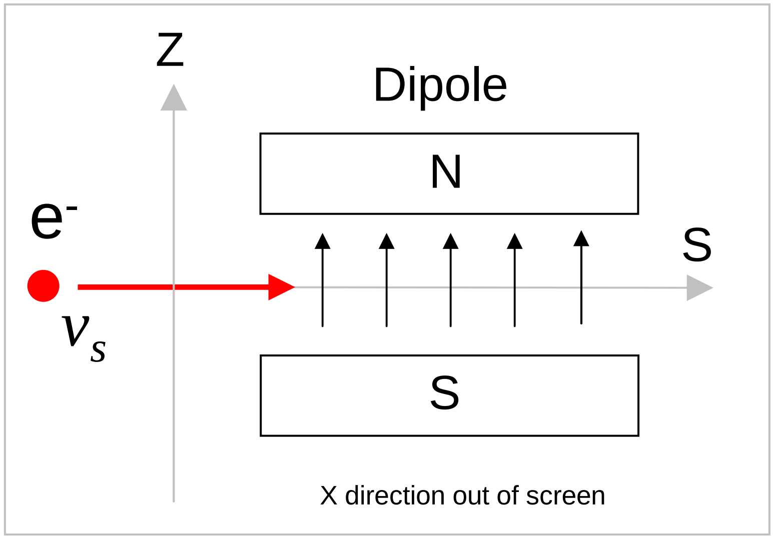
1. Introductory Presentation
2. Question Session

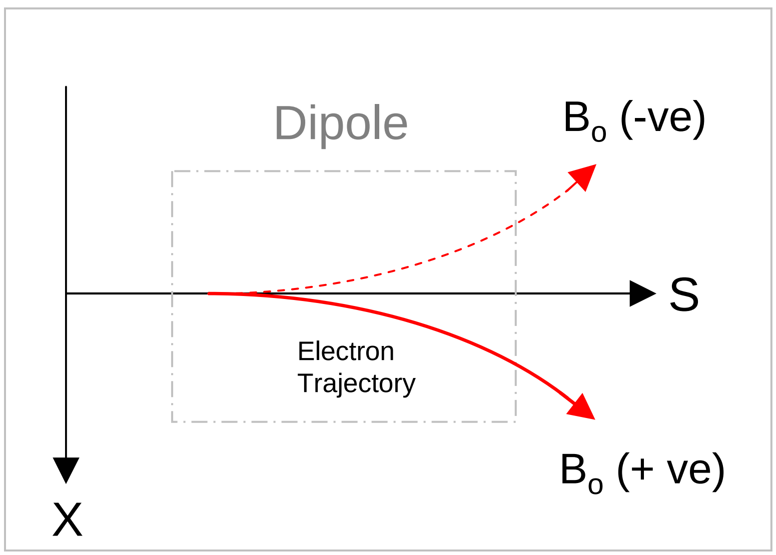
INTRODUCTORY MATERIAL IS SPLIT INTO 3 AREAS:

1. A Rough Guide to Undulators
2. The Finer Points of Undulator Radiation
3. Undulator Magnet Technology

PART 1

A ROUGH GUIDE TO UNDULATORS





Electron Motion In a Magnetic Field

A moving charge experiences a force when passing through a magnetic field, as described by the Lorentz force law.

Lorentz Force Law

$$\underline{F} = q(\underline{v} \times \underline{B}_\circ)$$

$$F_x = e(v_s B_\circ)$$

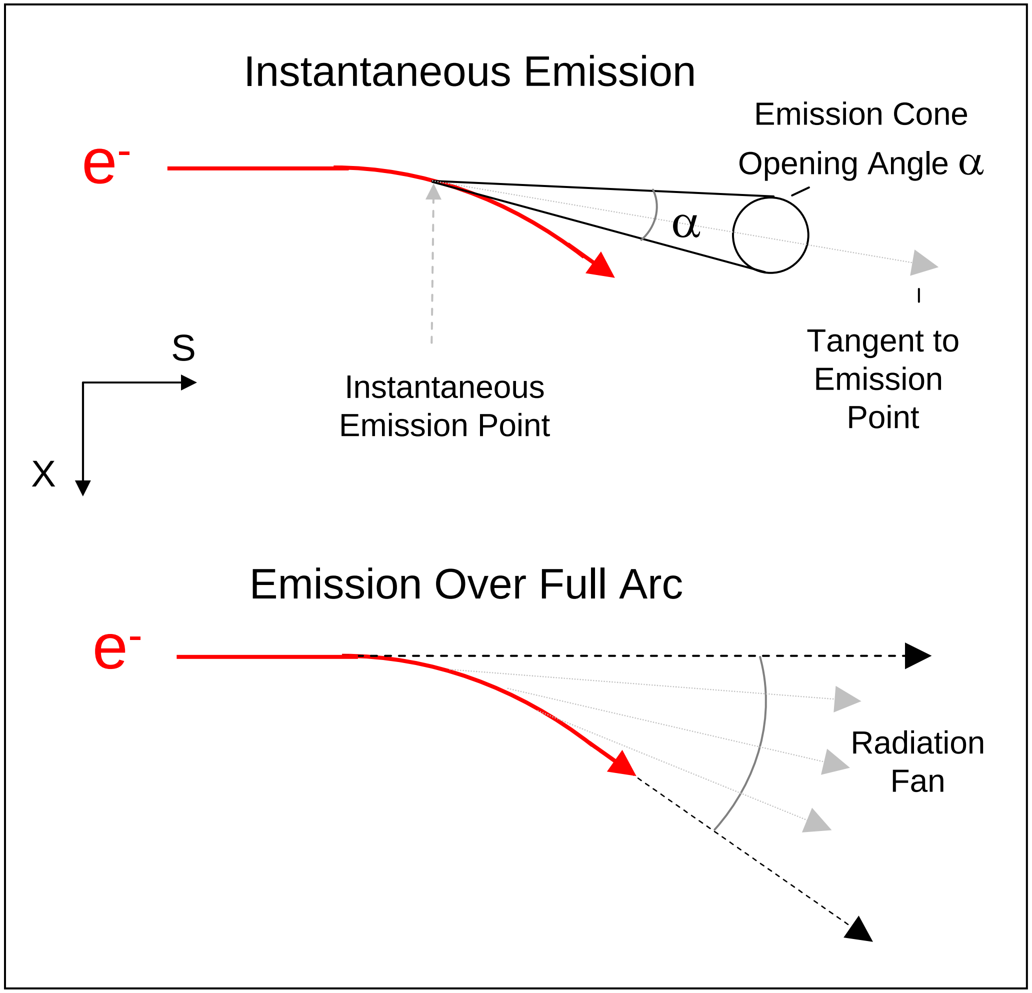
$$F_z = 0$$

$$F_s = -e(v_x B_\circ)$$

The electron will be deflected away from the S axis by the resulting acceleration, $a = F_x / m_e$.

Initially, $F_s = 0$ because $v_x = 0$, but as the deflection increases, v_x and hence F_s increases. F_s becomes important for reasons we shall see later on.

Emission of Radiation From a Moving Charge



Any charge that experiences an acceleration will radiate electromagnetic waves.

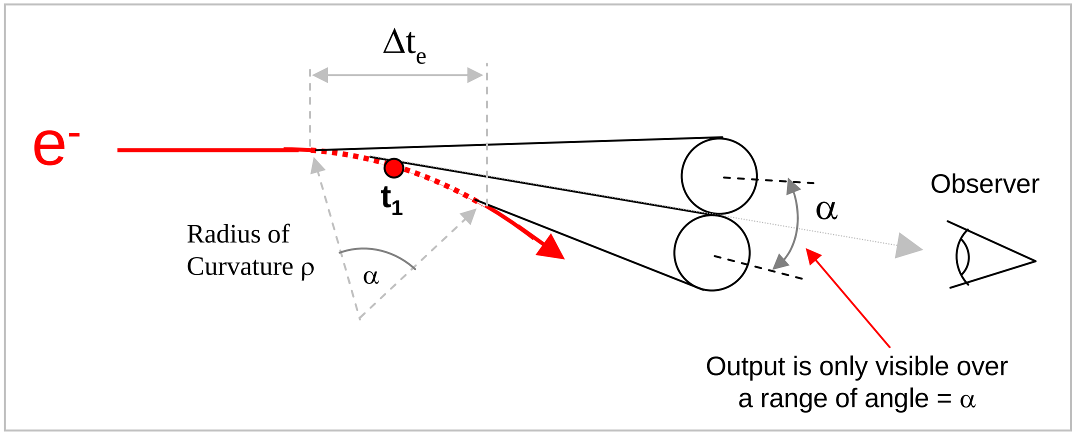
For this geometry the waves will be emitted along a tangent to the arc of motion.

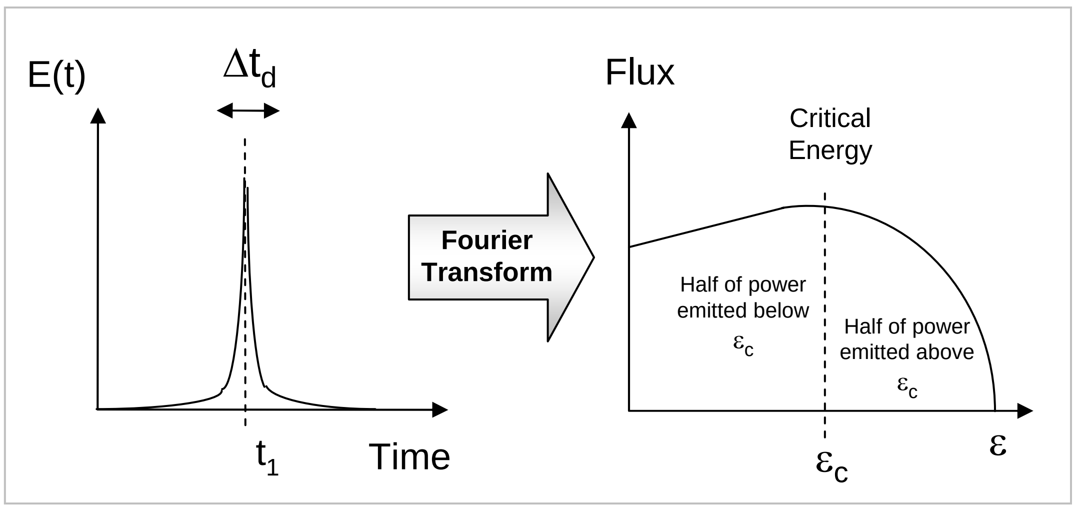
Due to the motion, an observer will see the waves emitted into a cone of angle α.

Over the full motion, the electron will sweep out a wide radiation fan.

This is 'bending magnet' radiation.

Radiation Characteristics





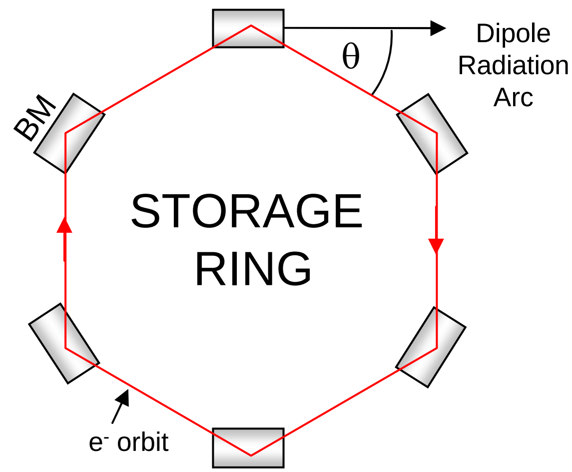
The observer will only 'see' the electric field, E(t), of the emission whilst the cone crosses the line of sight.

The observer will see a short pulse in the electric field as the electron sweeps by.

The photon emission rate (the Flux) at photon energy ε is obtained from $|E(t)|^2$ via a Fourier Transform (FT).

A property of the FT is that a short time signal produces a broad frequency response. Since the photon energy is $\varepsilon=\hbar\nu$, the short electric pulse produces a broad flux spectrum.

A ROUGH GUIDE TO UNDULATORS



Emitted Radiation has Characteristic Photon Energy

$$\varepsilon_c = 0.665\, B_\circ\, E^2$$

ε_c – Critical Photon Energy [keV]

E – Electron Energy in [GeV]

B_o – Magnetic Field in [Tesla]

Producing X-Rays

The characteristic energy of the emitted photons will be determined by the energy of the electron beam.

The most natural unit of energy when discussing the electron is the electron volt, eV. It is the energy received by an electron that is accelerated through 1 V.

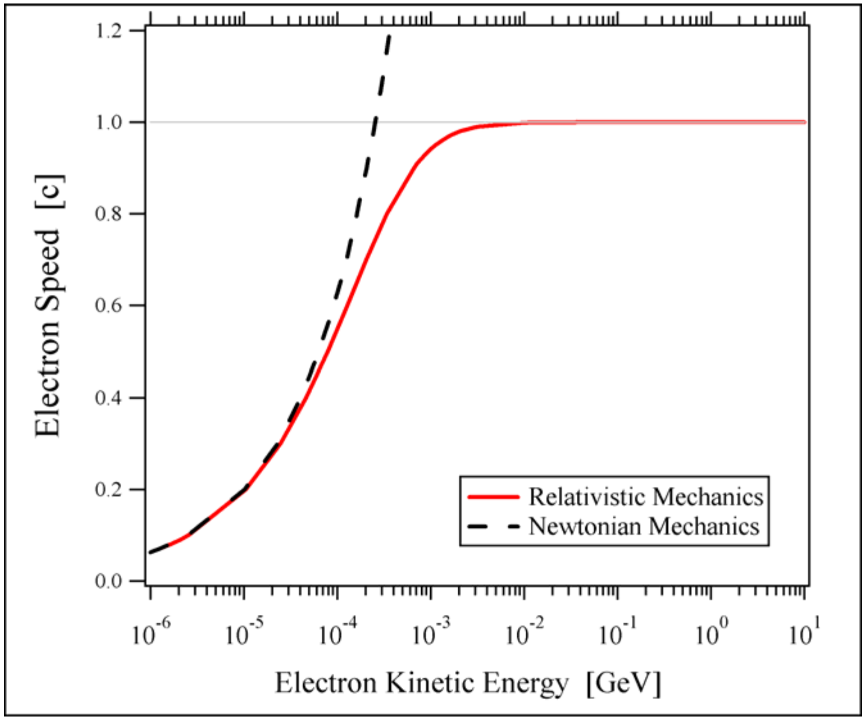
To reach the X-Ray part of the spectrum, we need to produce photons with a energy, $\varepsilon = \hbar\omega$, of the order of ~ keV.

X-Rays → ε ~ 1-100 keV photons

To achieve a photon energy ε_c ~ keV, we need an electron with an energy, E, of the order of GeV.

To reach this high electron energy, we construct a storage ring, with bending magnets to define the shape of the orbit.

A ROUGH GUIDE TO UNDULATORS



For the DLS storage ring E = 3GeV
Which gives γ = 5871

Hence the cone opening angle will be:

α = 0.17 mrad = 0.01°

Relativistic Electron Motion

An electron with an energy of ~GeV is extremely relativistic. The electron motion no longer obeys Newton's Laws.

The behaviour of the electron is described by the relativistic parameter γ, where;

$$\gamma = \frac{E}{E_o} = \frac{E}{m_e c^2} = 1957 E[GeV]$$

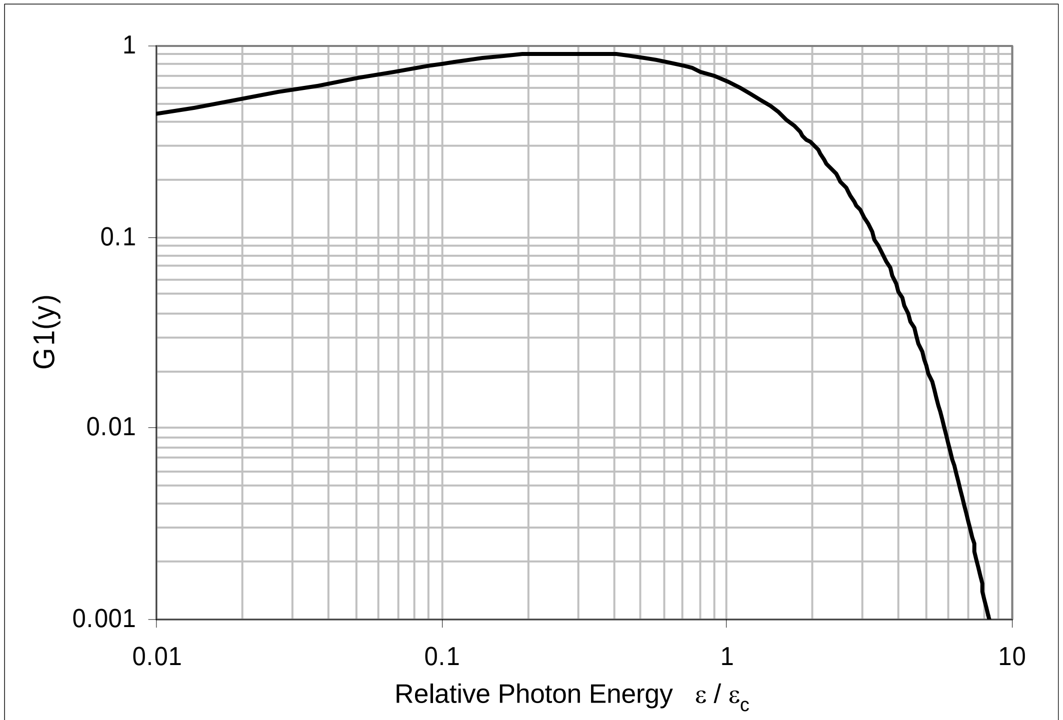
γ will determine the opening angle of the radiation cone:

$\alpha = 1/\gamma$ [radians]

γ is an important parameter, and appears in many equations connected with synchrotron radiation.

A ROUGH GUIDE TO UNDULATORS

The Universal Curve



Flux per unit [mrad] Horizontal angle, per 0.1% mono' relative bandwidth

Photon Flux Output

$$\frac{d\Phi}{d\theta} = 2.457 \times 10^{13}\, E\, I\, G_1(y)$$

Beam Current [Amps]

Radiation sources are graded by the number of photons emitted at a particular energy, per second.

The standard units of photon flux are:

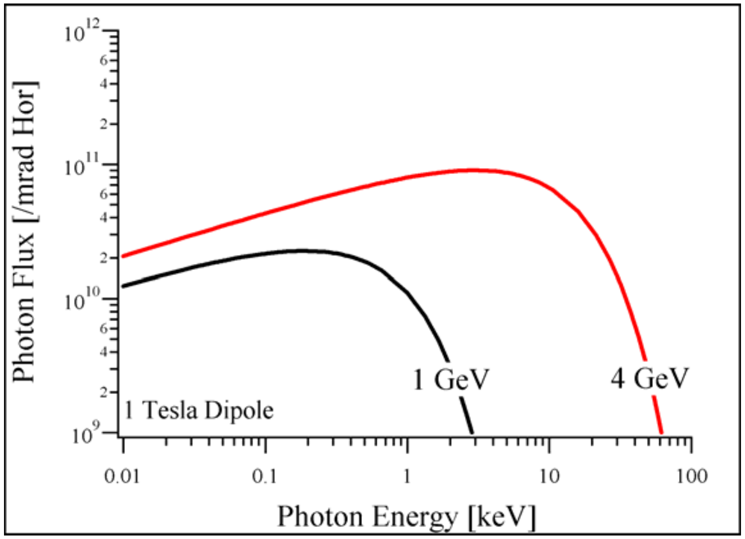
[ph / s / 0.1% relative bandwidth]

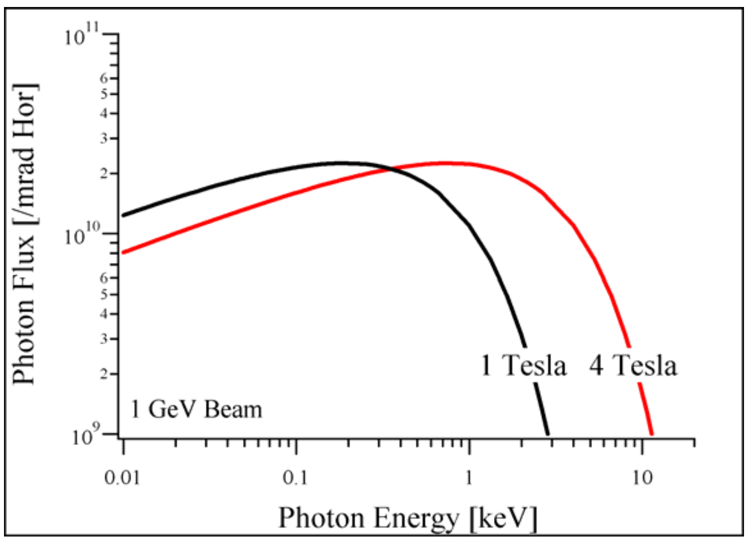
This allows us to account for the effect of monochromator used to select the energy ε. We obtain the flux *at the sample,* i.e. after the cone has passed through the monochromator.

The 'Universal Curve' allows us to estimate the flux obtained from any dipole magnet in any storage ring, from knowledge of its critical energy.

The curve gives the flux per mrad of Horizontal fan width accepted, and all of the vertical fan (~1/ γ).

A ROUGH GUIDE TO UNDULATORS





Output Optimisation

We now look at a few examples of how we can adjust the photon output of the bending magnet (BM).

By altering either the electron energy or the magnetic field, we can increase the flux output, and shift the critical energy.

We can therefore tailor the radiation output to suit our experimental needs

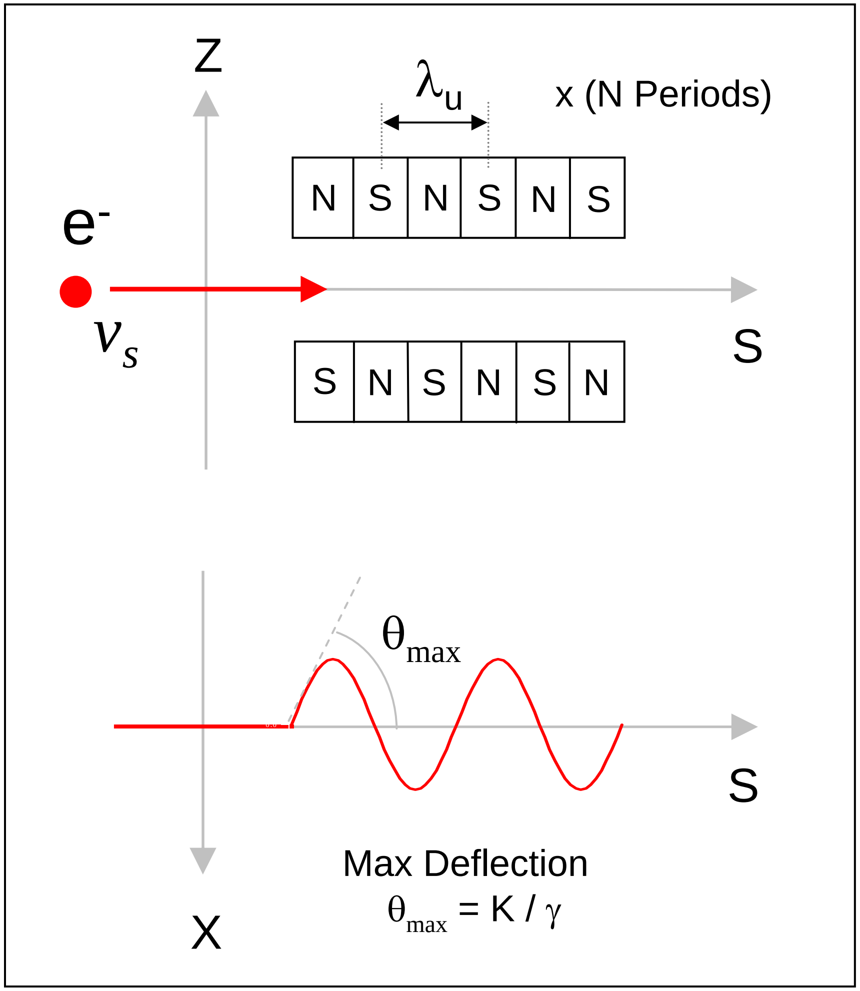
However, we cannot simply insert a BM with arbitrary properties into the ring, as the electron beam would crash into the walls of the storage ring.

How can we achieve a tuneable high photon energy source on a low/medium electron energy ring?

We need a magnetic array that doesn’t disturb the beam orbit.

A ROUGH GUIDE TO UNDULATORS

The Insertion Device



We produce our prototype undulator by constructing an alternating array of bending magnets. The structure has a magnetic periodicity of λ_u with N periods in total. The electron exits the array with the same angle and transverse position with which it entered.

The electron takes a sinusoidal path, with a max angular deflection given by K/γ, where K is the deflection parameter given by;

$$K = 0.0934\ \lambda_u\,[mm]\ B_o\,[T]$$

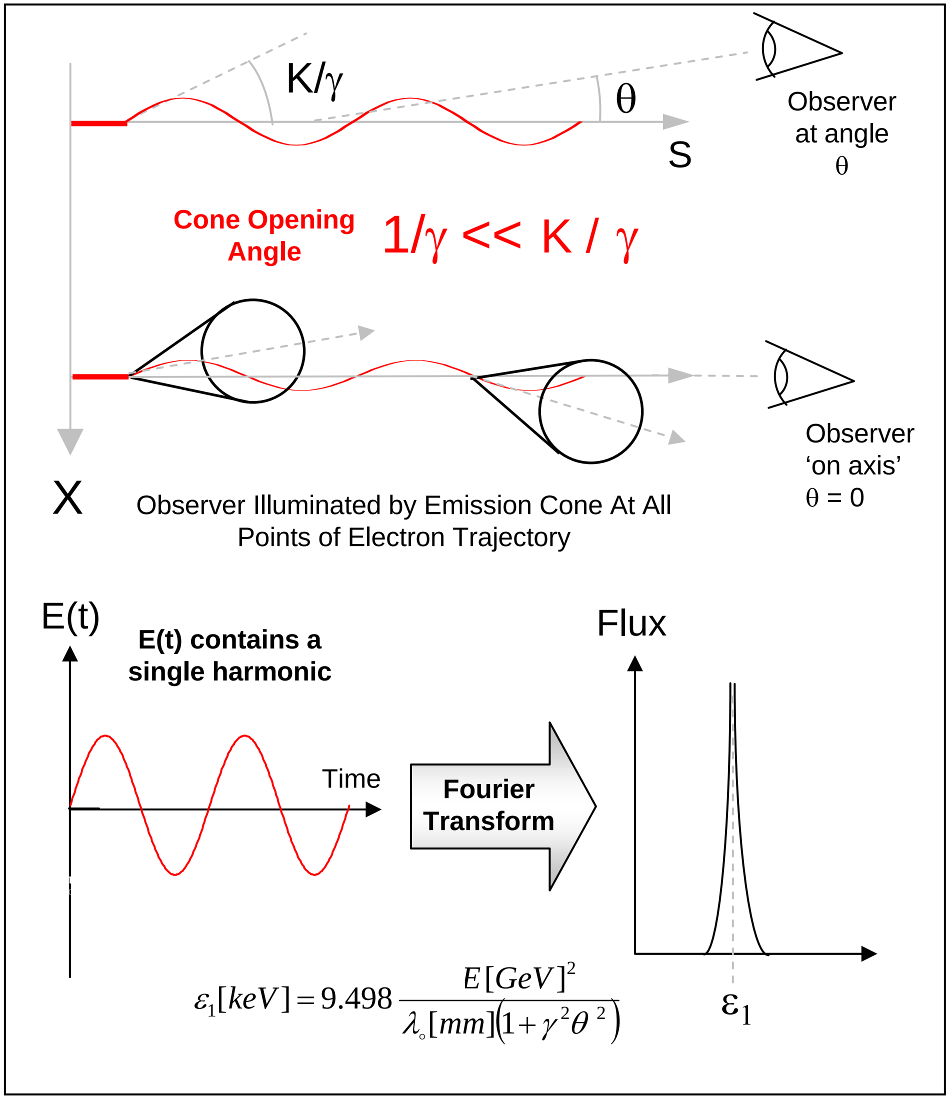
We will investigate the effect the sinusoidal motion has on the spectrum we obtain. Will it retain the characteristics of bending magnet radiation?

Consider 2 Cases:

K << 1: V.Low fields and V.Short λ_u

K ~ 1: Low fields and Short λ_u

A ROUGH GUIDE TO UNDULATORS



Case 1: K << 1

The max angular deflection is much less than the cone opening angle.

The observer will now 'see' the full sinusoidal variation of the electron trajectory. We would expect light to be emitted with a wavelength $\lambda_r = \lambda_u$.

However relativistic effects will considerably shorten this.

In the electron Frame of Reference:

Lorentz Contraction $\lambda_u \rightarrow \dfrac{\lambda_u}{\gamma}$

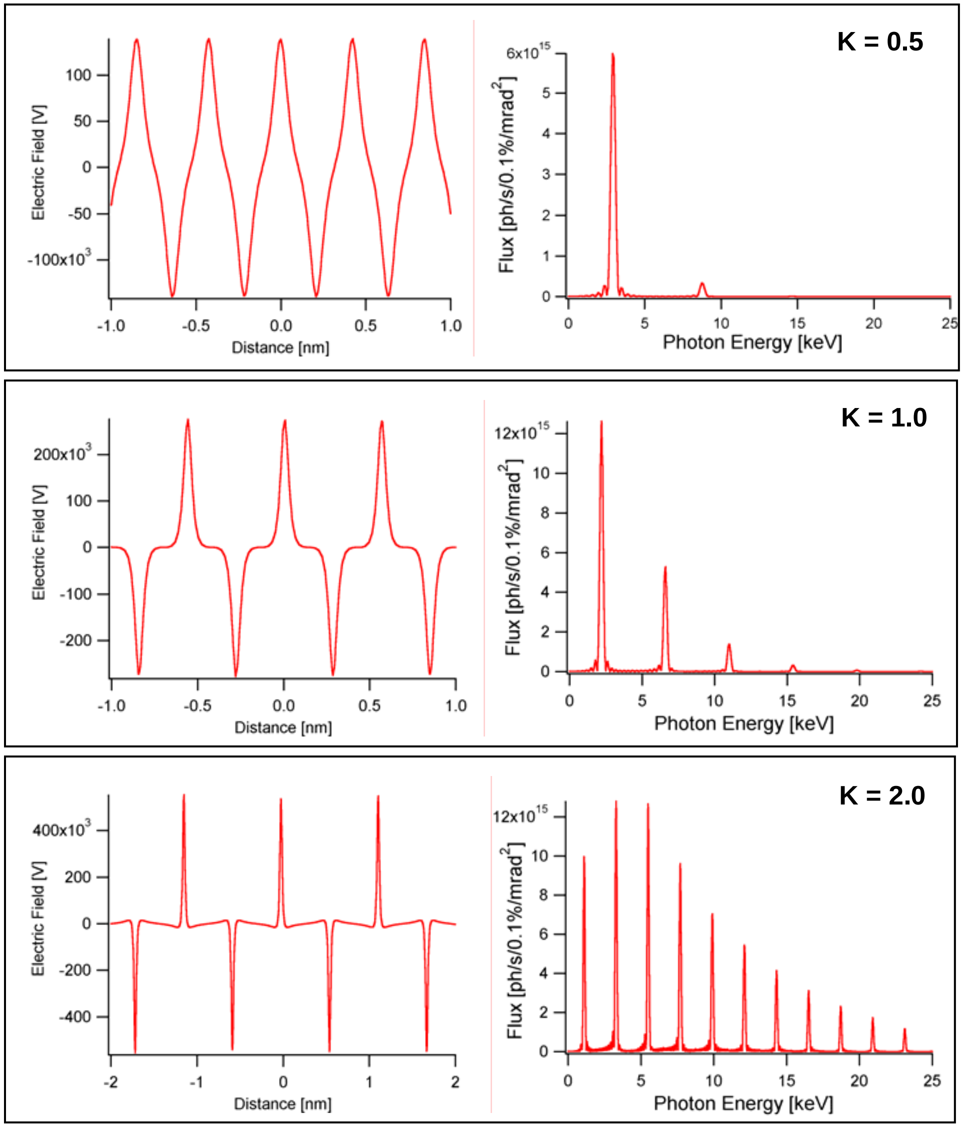
Doppler Effect $\lambda_u \rightarrow \dfrac{\lambda_u}{2\gamma}(1+\gamma^2\theta^2)$

Combined Effect $\lambda_r \rightarrow \dfrac{\lambda_u}{2\gamma^2}(1+\gamma^2\theta^2)$

For ~GeV machines, $2\gamma^2 \sim 10^7$

Hence $\lambda_u \sim$ mm $\rightarrow$ $\lambda_r \sim$ Å

A ROUGH GUIDE TO UNDULATORS



Case 2 K ~ 1: The Undulator

The electron motion is not only sinusoidal along X, but also along S. As K increases, we can no longer neglect the longitudinal ‘undulating’ motion caused by F_s.

This is responsible for the introduction of the higher harmonics into the spectrum.

Due to the E field profile, we only see odd harmonics ‘on axis’. Even harmonics peak far off axis, and are less useful.

The photon energy of the n^{th} harmonic depends on the value of K, E, and λ_u, and also the angle of observation θ.

Photon Energy

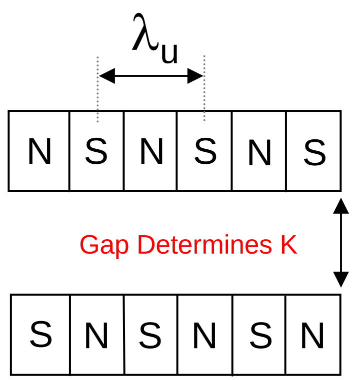
$$\varepsilon_n = 9.498 \frac{n E^2}{\lambda_u \left(1 + \frac{K^2}{2} + \gamma^2 \theta^2\right)}$$

PART 2

THE FINER POINTS OF UNDULATOR RADIATION

UNDULATOR RADIATION: THE FINER POINTS

Undulator Design



1. Choose value of λ_u.
2. Set K by altering the gap.

Undulator Parameters.

We want to know the performance (i.e. the photon flux) of our device for a given set of parameters, λ_u, K and N.

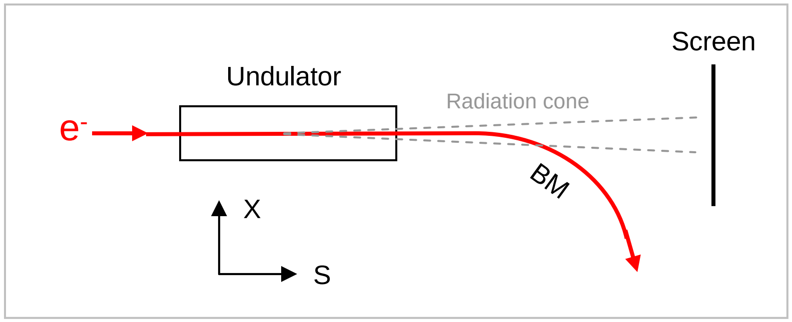
The way the device is built is that a value of λ_u is chosen, by arranging the length of the magnet blocks, and the value of K is set by varying the separation of the upper and lower magnet arrays.

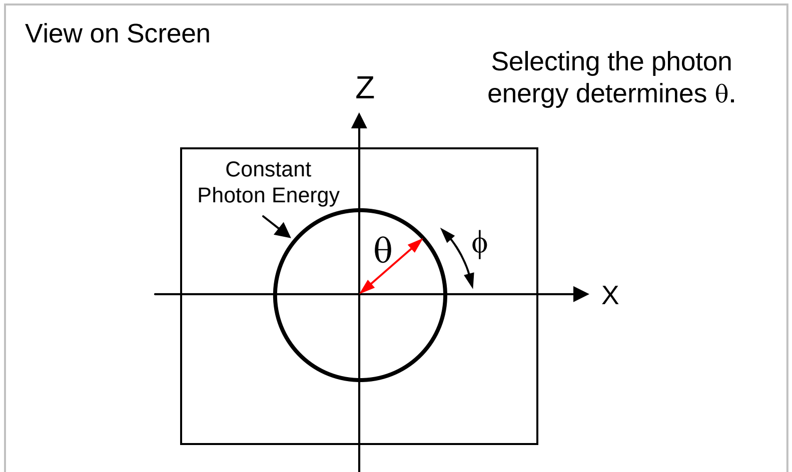
So for a fixed value of λ_u, we want to know how the flux output of the device varies with K.

Ultimately, the experimenter is not interested in the parameters of the device, they only want to know the flux output at a given photon energy.

So for a final comparison of parameters, we want to know the flux output of a given device (fixed λ_u, and variable K) as a function of the photon energy ε.

UNDULATOR RADIATION: THE FINER POINTS





$$\varepsilon_n = 9.498 \frac{nE^2}{\lambda_u \left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)}$$

Emission Profile

Imagine a screen placed in the path of the undulator beam.

The resulting spot pattern can be decomposed into contributions from each harmonic. We know that for the n^{th} harmonic (from the equation for ε_n) that the photon energy will decrease from the on axis value as θ increases.

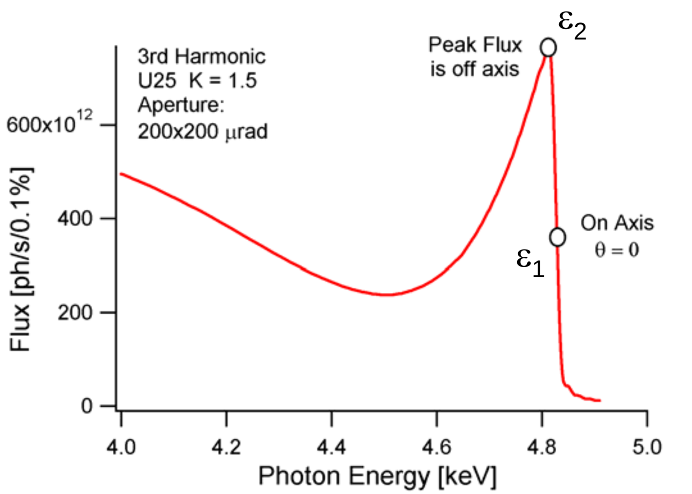
The max energy is in the centre of the screen (‘on axis’).

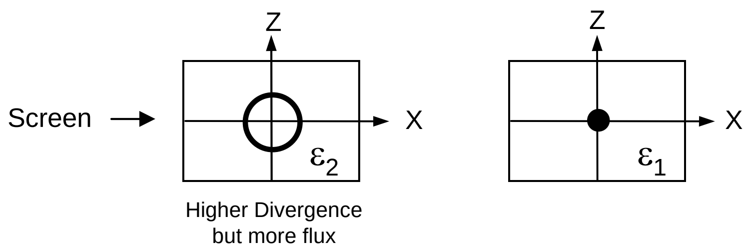
We also see that for the n^{th} harmonic, lines of constant photon energy are also lines of constant θ – i.e. circles.

If we pick out a single photon energy from the beam, it will form a ring with an angular thickness (and hence energy spread) determined by the number of periods N.

UNDULATOR RADIATION: THE FINER POINTS

Total Flux for U25





Peak Flux at Photon Energy:

$$\varepsilon_n^{peak} = \left(1 - \frac{1}{Nn}\right)\varepsilon_n\,(\theta = 0)$$

Obtaining The Total Flux

When we perform an experiment, we select a single energy to work with. We want to know the performance of the device at this energy.

We ‘count’ the total number of photons hitting the screen for each photon energy (i.e. the number of photons landing on the circular band of radius θ, per second).

This will give us the total flux as a function of the photon energy. This allows us to estimate the performance of our device over our energy range of interest, and is vital for designing our undulator.

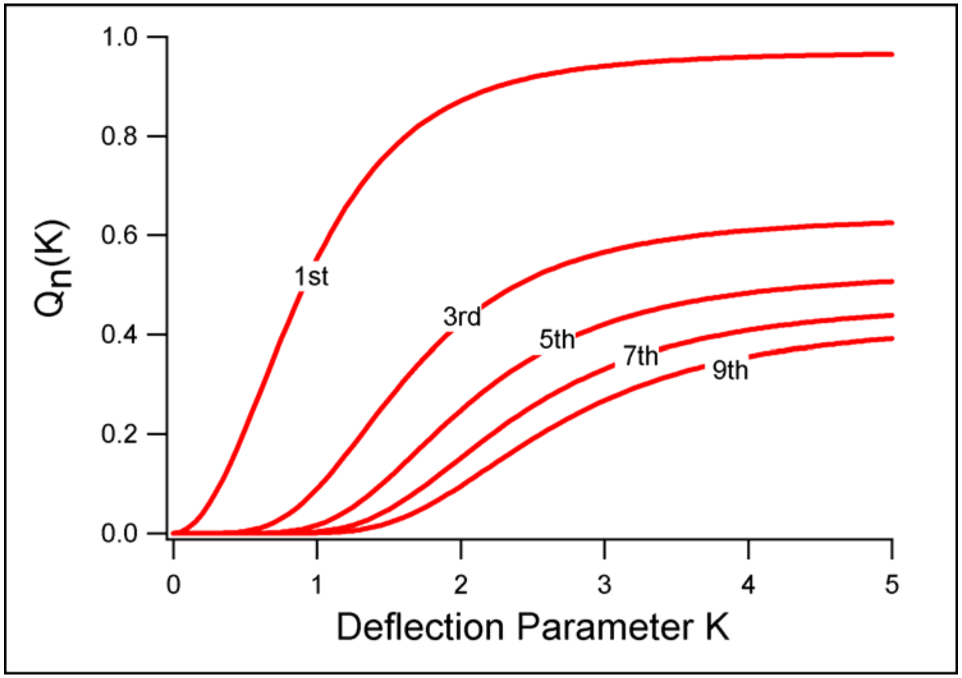
We find that at the max photon energy ($\varepsilon_n(\theta=0)$), we do not obtain the max flux.

We obtain the max flux at the slightly lower photon energy ε_n^{peak}.

We work with $\varepsilon_n(\theta=0)$), rather than ε_n^{peak}.

UNDULATOR RADIATION: THE FINER POINTS

The Q Function



Harmonic Flux Output

$$\Phi_n(K) = 1.4 \times 10^{14} \, N \, I \, Q_n(K)$$

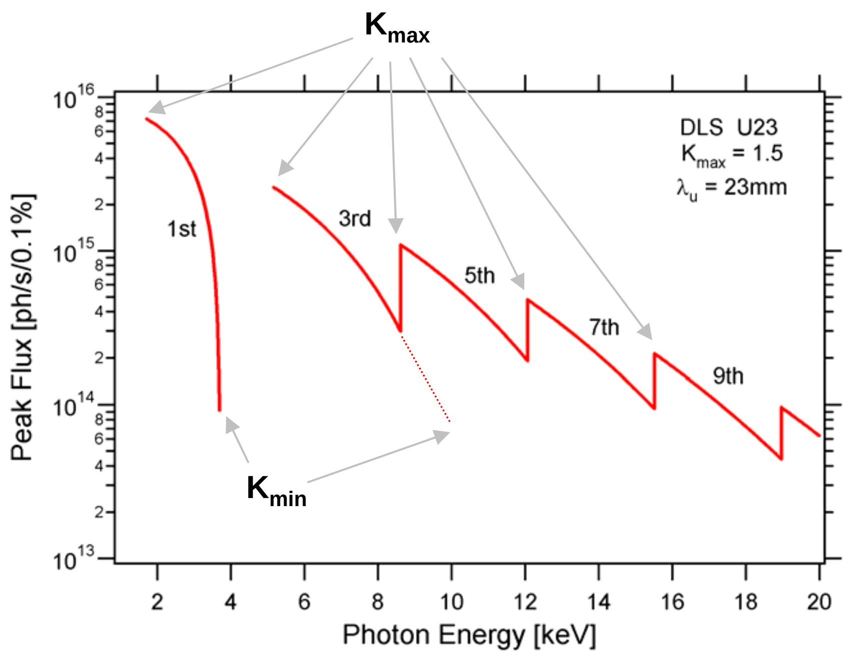
The Q function gives the *peak* flux (i.e. at ε_n^{peak}) expected on the n^{th} harmonic as a function of K. The units are again [ph/s/0.1%b.w.]

It is important to notice that λ_u does not influence the peak flux.

λ_u only plays a part in determining the photon energy: For a given K, reducing the value of λ_u will yield higher and higher photon energies. To reach the highest energies, we need undulators with a small λ_u.

The value of K alone allows us to predict the max flux we expect from our device. However, we need to know λ_u in order to calculate what photon energy it corresponds to.

UNDULATOR RADIATION: THE FINER POINTS



For energies just below 8.5 keV. the 3rd harmonic gives the most flux. However, above 8.5keV, it becomes more profitable to use the 5th. Similarly at 12keV for the 7th.

The only the part of the harmonic giving the maximum flux is displayed.

Undulator Performance

We want an estimate of the peak flux as a function of ε, not K. This can be obtained from $Q_n(K)$.

We calculate for each harmonic, the pair of numbers;

$$\Phi_n(\varepsilon) = \{\varepsilon_n^{peak}(K,\lambda_u), 1.4\times10^{14}\, I\, N\, Q_n(K)\}$$

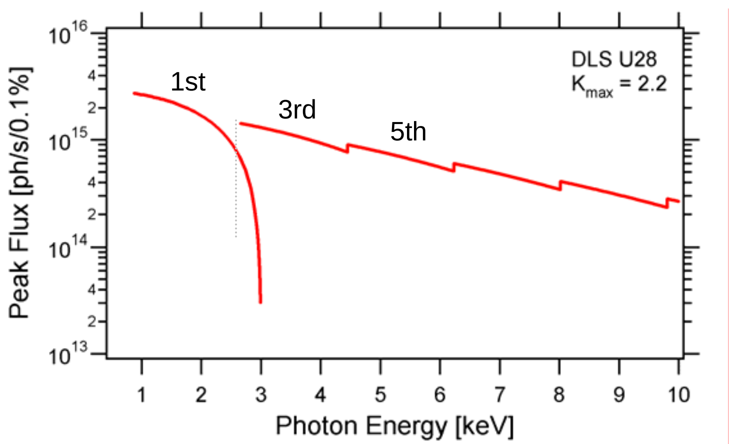
An undulator is designed to operate over a range of K. The flux performance can be plotted over this K range, to assess the suitability of the design.

From $\Phi_n(\varepsilon)$ we obtain the performance of the device over its full photon energy range.

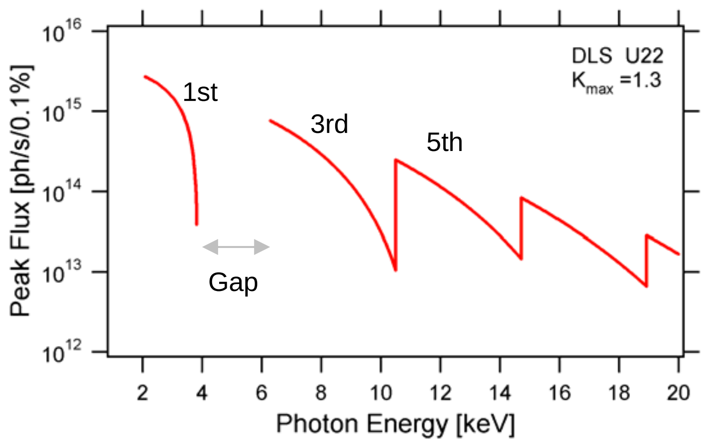
We only need to know K_{max}, λ_u and the number of periods in the device, N, in order to obtain the full performance of the device.

UNDULATOR RADIATION: THE FINER POINTS

Overlap over all harmonics Kmax > 2.2



Overlap over 3rd + higher harmonics Kmax > 1.3



Tuneability

Many experimental stations require a radiation source that is continuously tuneable over a wide range of energies.

To achieve this, we need to ensure that there are no gaps in the $\Phi_n(\varepsilon)$ function for our prospective device parameters.

The tuneability of the device is determined by K_{max}.

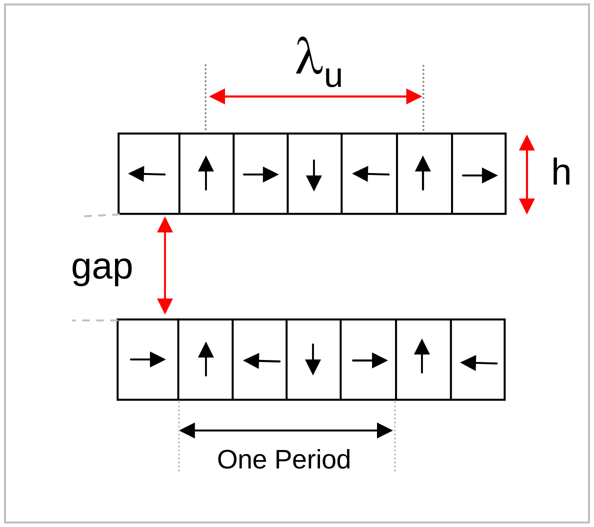
There are two practical choices:

1. Over lap over all adjacent harmonics. Ensure K_{max} >2.2
2. Overlap over the 3rd and all higher adjacent harmonics. Ensure K_{max} > 1.3.

PART 3

UNDULATOR TECHNOLOGY

UNDULATOR TECHNOLOGY



Device Performance

$$K = 0.168\, B_r \lambda_u\, e^{-\frac{\pi \lambda_u}{gap}}$$

B_r – Remanent Field of permanent magnet material (a measure of how magnetic the material is).

All lengths in [mm]. B_r in [Tesla]
(K expression assumes $h > \lambda_u/2$)

Magnetic Structure

To create our sinusoidal field, we use an array of permanent magnet blocks.

Electromagnetic coils are sometimes used, but these are only usually competitive for Wiggler devices.

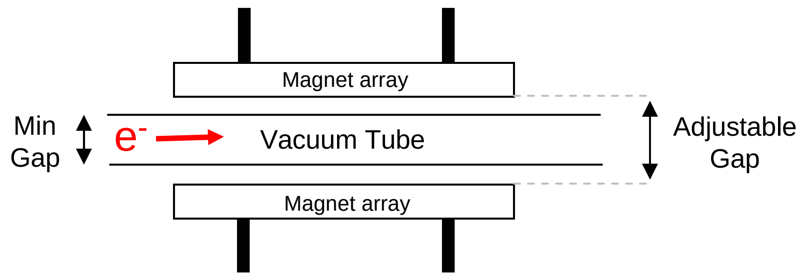
Instead of using 2 blocks per period (N, S, N, S,…), we find that a much better sinusoidal field is produced by using 4 blocks, and 'rotating' the field vector by 90° on each block.

The challenge for the technology is producing a high field (and hence K_{max} value) for a small λ_u. The difficulty arises because a small λ_u means less magnetic material to physically produce the required field.

The magnetic performance depends on the ratio of λ_u / gap. It is easier to produce a high field with either a large λ_u or a small gap. The min gap is set by the machine parameters.

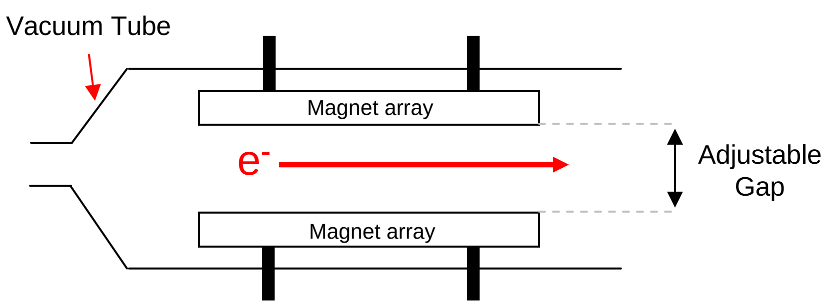
UNDULATOR TECHNOLOGY

Out Of Vacuum Undulator



The minimum gap is determined by the diameter of the vacuum tube.

In Vacuum Undulator



The minimum gap is determined by the dimensions of the electron beam

Device Types

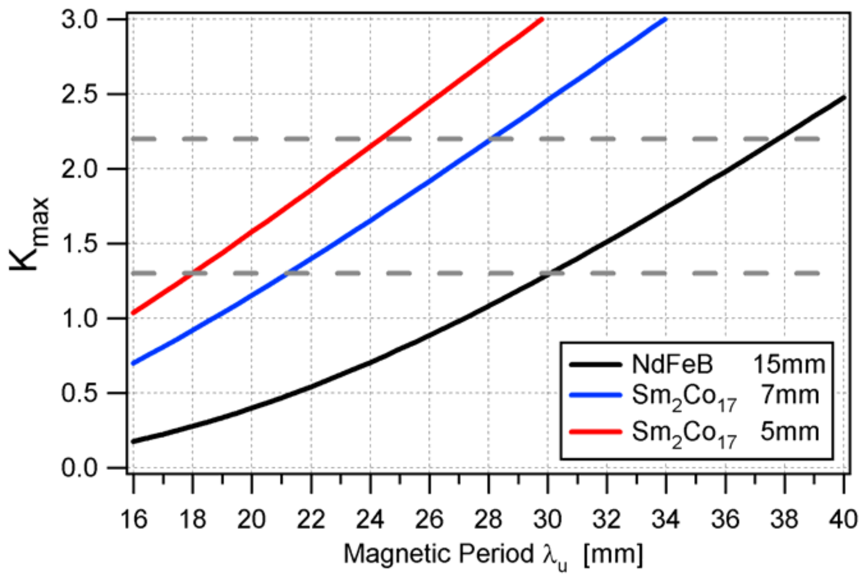
There are two approaches to the mechanical arrangement of the upper and lower arrays

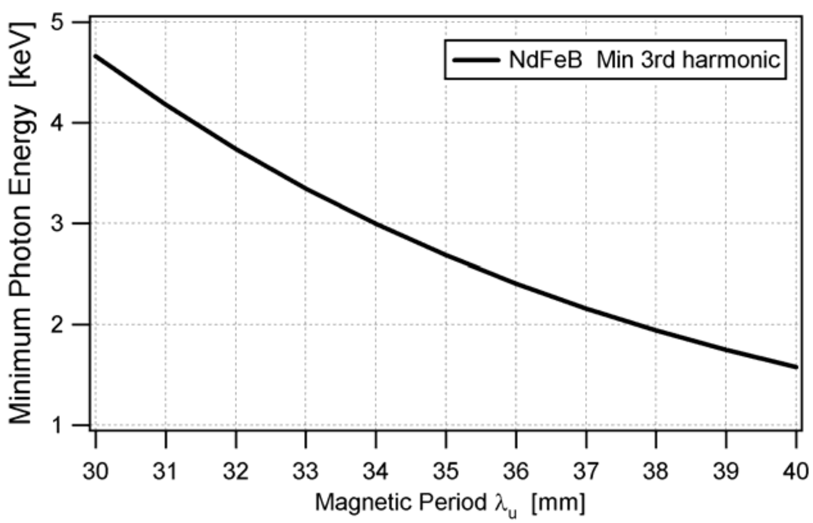
1. Position the arrays above and below the vacuum tube (out of vacuum device).

2. Position the whole magnetic assembly inside the vacuum tube (in vacuum device).

The advantage of the in vacuum device is that it allows for a much smaller minimum gap, and hence larger K_{max} for a given λ_u. This gives it the advantage at high photon energies.

UNDULATOR TECHNOLOGY





Magnetic Material

We have the choice of two types of magnet material.

NdFeB: High field (Br = 1.3T), low radiation + low heat resistance.

Sm_2Co_{17}: Lower field Br = 1.03T, but good radiation and heat resistance.

For the out of vacuum, we choose NdFeB for its high field. For the in vacuum, we choose Sm_2Co_{17}, due to its higher radiation resistance.

To assess the relative merits of these materials, we plot K_{max} vs λ_u. For each material, the value of λ_u determines the value of K_{max}.

Ensuring complete overlap is not the only requirement. The device must also guarantee that the beamline can support the minimum working energy. This may mean increasing λ_u.

THE
END

(now design your own undulator…)